Implementation of Soil-Structure Interaction Models in Performance Based Design Procedures

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A soon to be published guidelines document for the design of seismic retrofits for existing buildings is based on performance-based design principles as implemented through so-called nonlinear static procedures (NSPs). In these procedures, the global inelastic deformation demand on the structure is computed from the response of an equivalent nonlinear single-degree-of-freedom (SDOF) system, the response of which is estimated from that of an elastic SDOF system. The guidelines were developed as part of the *ATC-55* project, which is summarized by Comartin (this conference). The objective of the present paper is to describe one component of the *ATC-55* project related to the implementation of soil-structure interaction (SSI) principles into NSPs. SSI effects are most important at short periods (i.e., T less than approximately 0.5 s). Three SSI phenomena can contribute to NSPs. First, flexibility at the soil-foundation interface can be incorporated into nonlinear pushover curves for the structure. These foundation spring models were incorporated into NSPs that pre-existed the *ATC-55* project, and are not emphasized here. Second, SSI affects demand spectra through the effective system damping, which is the damping ratio for which spectral ordinates should be calculated. Third, kinematic SSI reduces ordinates of the demand spectra. This paper describes how damping and kinematic SSI effects have been incorporated into the recommended seismic analysis procedures for existing buildings.

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1.0 INTRODUCTION

In this paper we present simplified procedures for including the effects of interaction between a structure and the supporting soils in nonlinear inelastic seismic analyses. The procedures described here were developed as part of the *ATC-55* project and will be formally presented in *FEMA-440* (2004).

There are three primary categories of soil-structure interaction (SSI) effects. These include: introduction of flexibility to the soil-foundation system with resulting lengthening of the system's fundamental response period (flexible foundation effects); filtering of the character of ground shaking transmitted to the structure (kinematic effects); and dissipation of energy from the soil-structure system through radiation and hysteretic soil damping (foundation damping effects). Current analysis procedures in *FEMA 356* (2000) and *ATC 40* (1996) partially address the flexible foundation effect in guidelines for including the stiffness and strength of the geotechnical components of the foundation in the structural analysis model. However, those procedures do not address reduction of the shaking demand on the structure relative to the free field motion due to kinematic interaction or the foundation damping effect. Guidelines on including those effects in nonlinear inelastic analyses were introduced in *FEMA-440* and are summarized here. More detailed information can be found in Appendix 8 of *FEMA-440* (2004).

2.0 KINEMATIC INTERACTION EFFECTS

Kinematic interaction results from the presence of stiff foundation elements on or in soil, which causes foundation motions to deviate from free-field motions as a result of base slab averaging and embedment effects. The base slab averaging effect can be visualized by recognizing that the motion that would have occurred in the absence of the structure within and below the footprint of the building is spatially variable. Placement of a foundation slab across these variable motions produces an averaging effect in which the foundation motion is less than the localized maxima that would have occurred in the free-field. The embedment effect is simply associated with the reduction of ground motion that tends to occur with depth in a soil deposit.

This section covers simple models for the analysis of ground motion variations between the free-field and the foundation-level of structures. In general, these models must account for base slab averaging and embedment effects. Kinematic interaction for pile-supported foundations is not covered. Theoretical models for kinematic interaction effects are expressed as frequency-dependent ratios of the Fourier amplitudes (i.e., transfer functions) of foundation input motion (FIM) to free-field motion. The FIM is the theoretical motion of the base slab if the foundation and structure had no mass, and is a more appropriate motion for structural response analysis than is the free-field motion.

In the following sections, formulations for transfer functions that account for base slab averaging and embedment effects are presented. Recommendations are then provided regarding how transfer functions can be used to modify a free-field response spectrum to estimate the FIM spectrum for use in nonlinear static procedures.

2.1 SHALLOW FOUNDATIONS AT THE GROUND SURFACE

Base-slab averaging results from inclined or incoherent incident wave fields. In the presence of those wave fields, translational base-slab motions are reduced relative to the free-field (rotational motions are also introduced, but are not considered here). The reductions of base-slab translation tend to become more significant with decreasing period. The period-dependence of these effects is primarily associated with the increased effective size of the foundation relative to the seismic wavelengths at low periods. In addition, ground motions are more incoherent at low periods.

Veletsos and co-workers (1989, 1997) developed useful models for theoretical base slab averaging that combine an analytical representation of the spatial variation of ground motion with rigorous treatment of foundation-soil contact. The transfer function amplitudes computed by the Veletsos group are presented in Figure 1 for circular and rectangular foundations subject to vertically incident, incoherent shear waves. Similar curves are available for other wave fields. The transfer functions in Figure 1 are plotted against the dimensionless frequency parameter $\tilde{a}_0$, defined as follows for circular and rectangular foundations subject to vertically incident waves, respectively,

$$\tilde{a}_0 = \kappa a_0 \text{ (circular); } \tilde{a}_o = \frac{\omega b_e \kappa}{2V_{s,r}} \text{ (rectangular),} \tag{1}$$

where $a_0 = \omega r/V_{s,r}$, $V_{s,r}$ = strain-reduced shear wave velocity, r = radius of circular foundation, $b_e = \sqrt{ab}$, $a \times b$ = full footprint dimensions of rectangular foundation, and κ = a ground motion incoherence parameter.

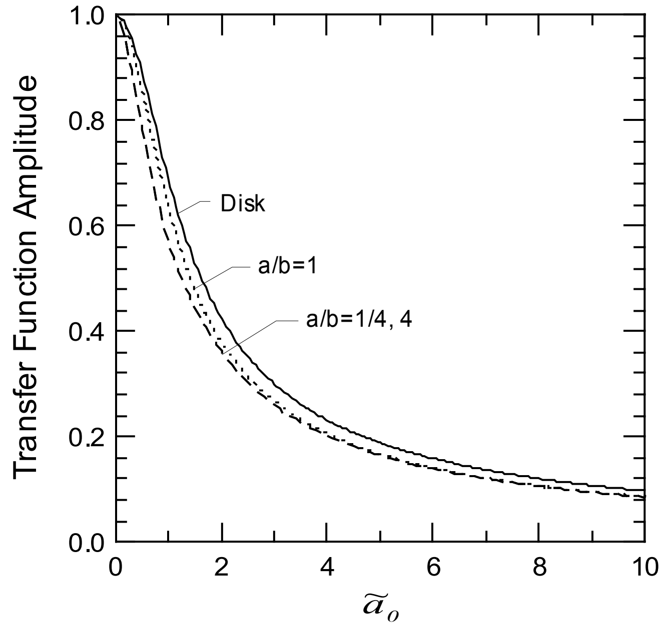

Figure 1. Amplitude of transfer function between free-field motion and FIM for vertically incident incoherent waves. Modified from Veletsos and Prasad (1989) and Veletsos et al. (1997).

Kim and Stewart (2003) calibrated Veletsos' analysis procedure against observed foundation / free-field ground motion variations as quantified by frequency-dependent transmissibility function amplitudes, $|H|$. Veletsos' models were fit to $|H|$ and apparent κ-values (denoted κ_a) were fit to the data. Those κ_a values reflect not only incoherence effects, but necessarily also include average foundation flexibility and wave inclination effects for the calibration data set. The structures in the calibration data set generally have shallow foundations that are inter-connected (i.e., continuous mats or footings inter-connected with grade beams). Parameter κ_a was found to be correlated to average soil shear wave velocity approximately as follows:

$$\kappa_a = -0.037 + 0.00074 V_s \text{ or } \kappa_a \approx 0.00065 V_s \tag{2}$$

where V_s = small strain shear wave velocity in m/s. The fact that κ_a is nearly proportional to V_s (Eq. 2) causes dimensionless frequency term $\tilde{a}_0$ to effectively reduce to a function of frequency and foundation size (b_e). This is shown by the following, which is written for vertically propagating waves ($\alpha_v = 0$):

$$\tilde{a}_0 = \frac{\omega b_e \kappa}{2V_{s,r}} \approx \frac{\omega b_e n_1 V_s}{2n_2 V_s} = \frac{\omega b_e n_1}{2n_2} \tag{3}$$

where $n_1 \approx 6.5 \times 10^{-4}$ s/m and n_2 is the square root of the soil modulus reduction factor, which can be estimated as shown in Table 1 (BSSC, 2001). In the remainder of this paper, n_2 will be taken as 0.65, which is the appropriate value for regions of high seismicity such as coastal California.

Table 1. Approximate values of n_2

	Peak Ground Acceleration (PGA)			
	0.10g	0.15g	0.20g	0.30g
n_2	0.90	0.80	0.70	0.65

Limitations of the model calibration by Kim and Stewart (2003), and hence the present approach, include: (1) foundations should have large in-plane stiffness, ideally a continuous mat foundation or interconnected footings/grade beams; (2) for non-embedded foundations, the foundation dimension should be less than 60 m unless the foundation elements are unusually stiff; (3) the approach should not be used for embedded foundations with $e/r > 0.5$; and (4) the approach should not be used for pile-supported structures in which the cap and soil are not in contact.

2.2 EMBEDDED SHALLOW FOUNDATIONS

Foundation "embedment" refers to a foundation base slab that is positioned at a lower elevation than the surrounding ground, which will usually occur when buildings have a basement. When subjected to vertically propagating coherent shear waves, embedded foundations experience a reduction in base-slab translational motions relative to the free-field.

Elsabee and Morray (1977) and Day (1978) developed analytical transfer functions relating base-slab translational motions to free-field translations for an incident wave field consisting of vertically propagating, coherent shear waves. Base-slab averaging does not occur within this wave field, but foundation translations are reduced relative to the free-field due to ground motion reductions with depth and wave scattering effects. Day's (1978) analyses were for a uniform elastic half space, while Elsabee and Morray's (1977) were for a finite soil layer. Results for both are shown together in Figure 3a for foundation embedment / radius ratio $e/r = 1.0$. The primary difference between the two solutions is oscillations in the finite soil layer case at high frequencies. Also shown in Figure 3a is the following approximate transfer function amplitude model developed by Elsabee and Morray (1977):

$$|H_u(\omega)| = \cos\left(\frac{e}{r} a_0\right) = \cos\left(\frac{e\omega}{V_s}\right) \geq 0.454 \tag{4}$$

where $a_0 = \omega r/V_s$ and e = foundation embedment. Figure 3b shows the transfer function amplitude model is a somewhat more convenient form in which it is plotted as a unique function of $\omega e/V_s$ (i.e., in this form there is no dependence on foundation radius).

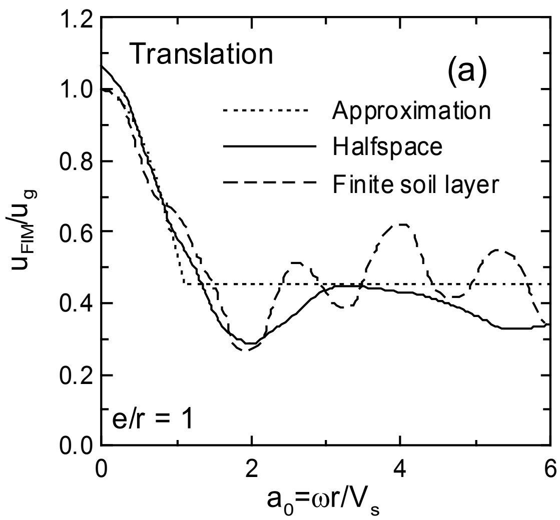

Figure 3a. Transfer function amplitudes for embedded cylinders from Day (1978) and Elsabee and Morray (1977) along with approximation

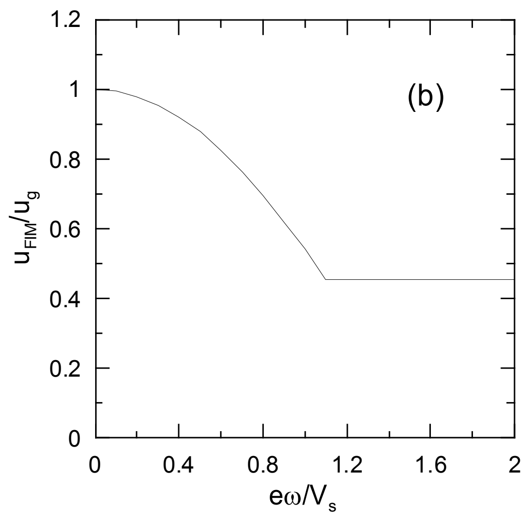

Figure 3b. Transfer function amplitude model by Elsabee and Morray (1977)

The results in Figure 3 can be contrasted with the behavior of a surface foundation, which would have no reduction of translational motions when subjected to vertically incident coherent shear waves. Transfer function amplitudes in the presence of more realistic incident wave fields can be estimated at each frequency by the product of the transfer function ordinates from Section 2.1 (for base slab averaging) and those from this section at the corresponding frequency.

The analysis procedure described herein has been verified against recorded motions from two relatively deeply embedded structures with circular foundations (Kim, 2001).

2.3 APPLICATION OF TRANSFER FUNCTIONS TO CALCULATION OF SPECTRAL ORDINATES OF FOUNDATION MOTIONS

Design-basis free-field motions are generally specified in terms of acceleration response spectra. The question addressed in this section is how this spectrum should be modified once the transfer function amplitude for the site has been evaluated using the analysis procedures described above.

When free-field motions are specified only as response spectral ordinates, the evaluation of a modified response spectrum consistent with the FIM is needed. Veletsos and Prasad (1989) evaluated ratios of foundation / free-field response spectral ordinates (at 2% damping) for conditions where the corresponding transfer function ordinates could be readily determined. The transfer function ordinates and ratios of response spectra (RRS) were compared for an input motion with specified power spectrum and random phase. The results indicated that transfer function ordinates provide a reasonable estimate of response spectral ratios for low frequencies (e.g, < 5 Hz), but that at high frequencies (≥ 10 Hz), transfer function ordinates are significantly smaller than response spectrum ratios. The inconsistency at high frequencies is attributed to the low energy content of free-field excitation at high frequencies and the saturation of spectral ordinates at these frequencies.

The analytical results of Veletsos and Prasad (1989) were checked by (1) calculating the transfer function for a fixed set of conditions (surface foundation, r = 50 m, V_s = 250 m/s), (2) using this transfer function to modify a set of recorded free-field time histories to corresponding foundation-level time histories, and (3) evaluating the RRS using the two time histories. The results are presented in Appendix 8 of *FEMA-440* (2004), and suggest that for ordinary ground motions that Veletsos's results summarized above are reasonable. However, it appears that some caution should be exercised for long-period ground motions such as those encountered on soft soil sites or for near-fault ground motions in the forward directivity region.

2.4 RECOMMENDED PROCEDURE

Based on the above, the following simplified procedure is recommended for analysis of kinematic interaction effects:

Step 1: Evaluate effective foundation size $b_e = \sqrt{ab}$, where a and b are the footprint dimensions (in feet) of the building foundation in plan view.

Step 2: Evaluate an RRS from base slab averaging (RRS_{bsa}) at the period of interest using Figure 4. The period that should be used in Figure 4 is the effective period of the foundation-structure system accounting for any lengthening due to foundation flexibility or structural yielding effects (denoted $\tilde{T}_{eq}$). An approximate equation to the curves in Figure 4 is presented below:

$$RRS_{bsa} = 1 - \frac{1}{14100}\left(\frac{b_e}{\tilde{T}_{eq}}\right)^{1.2} \quad (5)$$

Step 3: If the foundation is embedded a depth e from the ground surface, evaluate an additional RRS from embedment (RRS_e) at the period of interest using Figure 5. The period that should be used is the same as in Step 2. The equation of the curves in Figure 5 is,

$$RRS_e = \cos\left(\frac{2\pi\ e}{\tilde{T}_{eq} V_{s,r}}\right) \le \cos\left(\frac{10\pi\ e}{V_{s,r}}\right) \ge 0.454 \quad (6)$$

where $e =$ foundation embedment and $V_{s,r}$ = effective strain-degraded shear wave velocity in the soil. Factors that can be used to estimate $V_{s,r}$ from small-strain shear wave velocity V_s are given in Table 1.

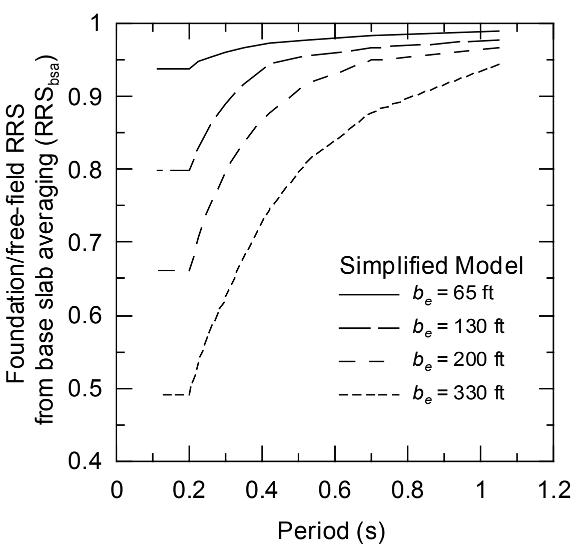

Figure 4. RRS_{bsa} from simplified model as function of foundation size, b_e

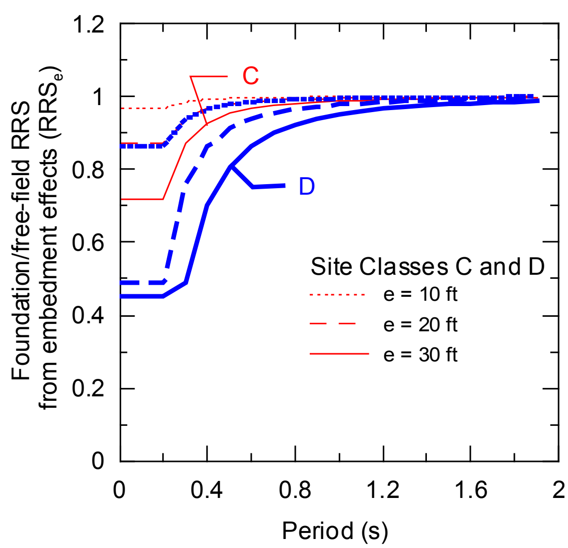

Figure 5. RRS_e for foundations with variable depths in NEHRP Site Classes C and D

Step 4: Evaluate the product of RRS_{bsa} and RRS_e to obtain the total RRS for the period of interest. The spectral ordinate of the foundation input motion at the period of interest is the product of the free field spectral ordinate and the total RRS.

Step 5: Repeat Steps 2 through 4 for other periods if desired to generate a complete spectrum for the foundation input motion.

3.0 FOUNDATION DAMPING

3.1 OVERVIEW

Inertia developed in a vibrating structure gives rise to base shear and moment at the foundation-soil interface, and these loads in turn cause displacements and rotations of the structure relative to the free field. These relative displacements and rotations are only possible because of compliance in the soil, which can significantly contribute to the overall structural flexibility. Moreover, the difference between the foundation input motion and free field motion gives rise to energy dissipation via radiation damping and hysteretic soil damping, and this energy dissipation affects the overall system damping. Since these effects are rooted in the structural inertia, they are referred to as *inertial interaction* effects, in contrast to the *kinematic interaction* effects discussed in Section 2.0.

Previous design documents (*FEMA-356*, 2000; *ATC-40*, 1996) contain provisions for evaluating the properties of foundation springs (e.g., Sections 10-3 and 10.4 of *ATC-40*), and hence this aspect of inertial interaction is not emphasized here. Rather, the *ATC-55* project examined the damping component of inertial interaction and the contribution of this damping to the overall system damping.

In the SSI literature, foundation stiffness and damping are often described in terms of an *impedance function*. The impedance function should account for the soil stratigraphy and foundation stiffness and geometry, and is typically computed using equivalent-linear soil properties appropriate for the in situ dynamic shear strains. Impedance functions can be evaluated for multiple independent foundation elements, or (more commonly) a single 6×6 matrix of impedance functions is used to represent the complete foundation (which assumes foundation rigidity).

A detailed discussion of impedance functions is presented in Appendix 8 of *FEMA-440* (2004). In simple terms, impedance functions can be thought of as springs and dashpots at the base of the foundation that accommodate translational and rotational deformations relative to the free-field. The coefficients that describe those springs and dashpots are frequency-dependent. At zero frequency (i.e., static loading), the springs stiffnesses are described by:

$$K_u = \frac{8}{2-\upsilon} G_{\max} r_u \, , \; K_\theta = \frac{8}{3(1-\upsilon)} G_{\max} r_\theta^3 \tag{7}$$

where subscript 'u' denotes translation and 'θ' denotes rotation in the vertical plane (sometimes referred to as rocking); G_{max} = small-strain soil shear modulus (can be calculated from V_s as $G_{max} = V_s^2\rho$, where ρ = mass density); ν = Poisson's ratio of soil; and radius terms r_u and r_θ are based on the area and moment of inertia, respectively, of the foundation as follows:

$$r_u = \sqrt{A_f/\pi} \, , \; r_\theta = \sqrt[4]{4I_f/\pi} \tag{8}$$

where A_f = foundation area and I_f = foundation moment of inertia. The dashpot coefficients that describe damping associated with translational and rotational vibrations (c_u and c_θ, respectively) are given by:

$$c_u = \beta_u \frac{K_u r_u}{V_S} \, , \; c_\theta = \beta_\theta \frac{K_\theta r_\theta}{V_S} \tag{9}$$

where β_u and β_θ are functions of frequency as shown in Figure 6. The curves in Figure 6 apply for a uniform soil medium of infinite depth (i.e., a halfspace) and a rigid, circular foundation at the ground surface.

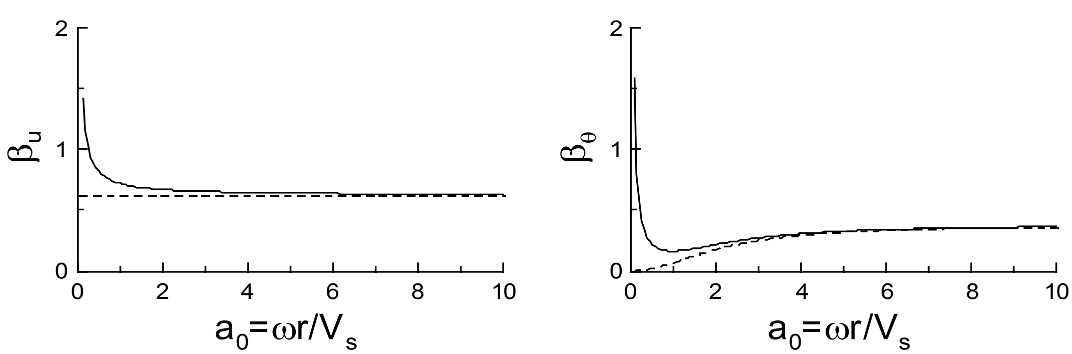

Figure 6. Foundation stiffness and damping factors for elastic halfspace (dotted line) and viscoelastic halfspace with 10% hysteretic soil damping. Poisson's ratio υ = 0.4. After Veletsos and Verbic (1973)

The combined effects of translational and rotational dashpots are often expressed by a foundation damping term β_f. The effects of foundation damping, in turn, on the response of a structure are represented by a modified damping ratio for the overall structural system. The initial damping ratio for the structure neglecting foundation damping is referred to as β_i, and is generally taken as 5%. The damping ratio of the complete structural system, accounting for foundation-soil interaction, as well as structural damping, is referred to as β_0. The change in damping ratio from β_i to β_0 modifies the elastic response spectrum. The spectral ordinates are reduced if $\beta_0 > \beta_i$.

The calculation of quantity β_0 is the objective of a foundation damping analysis. This quantity can be calculated from β_i and β_f using the following expression, which is modified from Jennings and Bielak (1973), Bielak (1975, 1976), and Veletsos and Nair (1975):

$$\beta_0 = \beta_f + \frac{\beta_i}{\left(\tilde{T}_{eq} / T_{eq}\right)^3} \tag{10}$$

where $\tilde{T}_{eq} / T_{eq}$ represents the period lengthening ratio of the structure in its degraded state (i.e., including the effects of structural ductility). Accordingly, the analysis of β_0 reduces to the evaluation of foundation damping β_f and period lengthening ratio $\tilde{T}_{eq} / T_{eq}$. The evaluation of these two quantities is described in the following sub-sections.

3.2 ANALYSIS OF PERIOD LENGTHENING TERM $\tilde{T}_{eq} / T_{eq}$

The period lengthening can be evaluated using the structural model employed in pushover analyses using the procedure that follows (this procedure remains under investigation, and may deviate slightly from what is ultimately recommended in *FEMA 440*):

1. Evaluate the first-mode vibration period of the model, including foundation springs. This period is $\tilde{T}$. This period is calculated using initial stiffness values (prior to yield of structural or soil spring elements).
2. Evaluate the first-mode vibration period of the model with the foundation springs removed (or their stiffness and capacity set to infinity). This period is T. As before, this period should correspond to pre-yield conditions.

3. Calculate the ratio $\tilde{T}/T$, which is the period lengthening under small-deformation (elastic) conditions.

4. Calculate $\tilde{T}_{eq}/T_{eq}$ using the following equation:

$$\frac{\tilde{T}_{eq}}{T_{eq}} = \left\{1 + \left(\frac{\Delta_f}{\Delta_s}\right)\left[\left(\frac{\tilde{T}}{T}\right)^2 - 1\right]\right\}^{0.5} \tag{11}$$

where Δ_f = average ductility of foundation springs (described further below) and Δ_s = target ductility level for design of superstructure (typically 2-4). For structures where the inertial interaction is dominated by rotation (as opposed to foundation translation), Δ_f can be calculated as the average ductility of the vertical foundation springs. As specified by *ATC 40* and *FEMA 356*, these springs are elastic-perfectly plastic, and the ductility of an individual foundation spring is simply the peak displacement normalized by the yield displacement. Alternatively, Δ_f can be approximated as $\Delta_f \approx (1/n_2)^2$. In many cases, $\Delta_f \approx \Delta_s$, and hence $\tilde{T}_{eq}/T_{eq} \approx \tilde{T}/T$.

3.3 ANALYSIS OF FOUNDATION DAMPING TERM β_F

Foundation damping term β_f is largest for stiff structures on soft soils, and decreases as the structure/soil stiffness decreases. Other critical factors include the aspect ratio of the structure (β_f decreases with the ratio of effective structure height to foundation radius, *h/r*) and the embedment ratio of the foundation (β_f increases with the ratio of foundation embedment to foundation radius, *e/r*). These factors that influence β_f also influence the period lengthening ratio of the structure. Hence, a convenient way to evaluate β_f is through direct relationships with $\tilde{T}_{eq}/T_{eq}$. The β_f - $\tilde{T}_{eq}/T_{eq}$ relationship in Figure 7 (left side) was derived for the condition described in Section 3.1, namely uniform soil and rigid, circular foundation at the ground surface. The relationship for e/r_u = 0.5 (right side) is a modification for embedment, the basis of which is described below in Section 3.3.2.

An approximate equation to the curves in Figure 7 is presented below for PGA > 0.2 g:

$$\beta_f = a_1\left(\left(\tilde{T}_{eq}/T_{eq}\right) - 1\right) + a_2\left(\left(\tilde{T}_{eq}/T_{eq}\right) - 1\right)^2 \tag{12}$$

where β_f is in percent and

$$a_1 = c_e \exp(4.5 - h/r_\theta),\ a_2 = c_e[25\ln(h/r_\theta) - 22],\ \text{and}\ c_e = 1.5(e/r_u) + 1 \qquad (13)$$

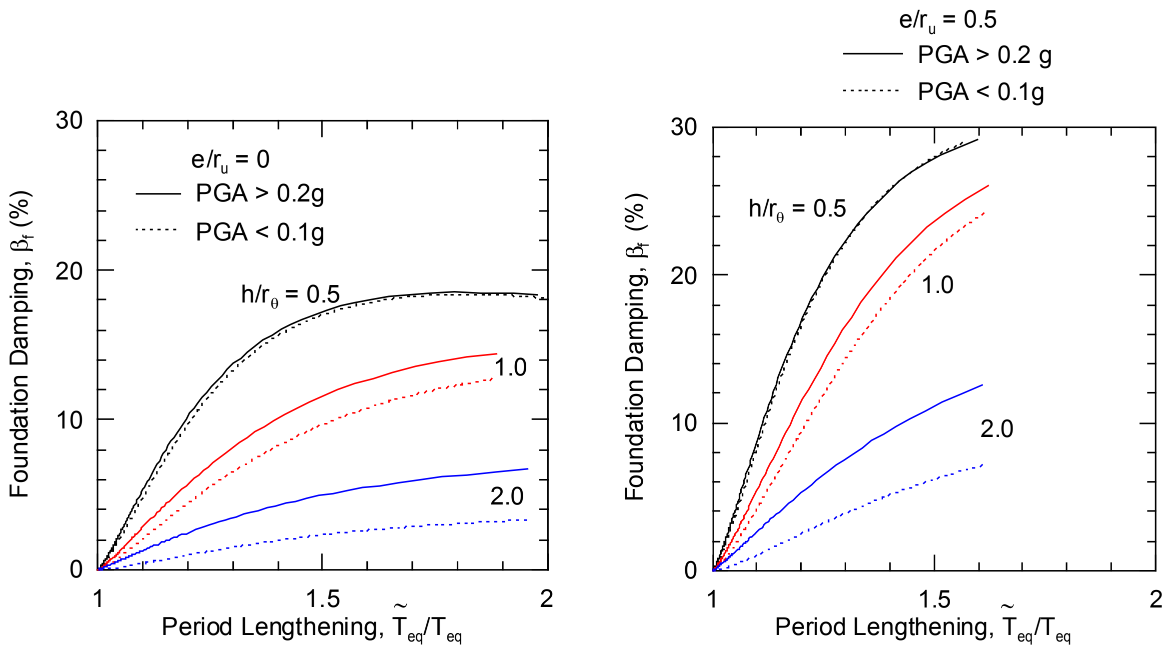

Figure 7. Foundation damping factor β_f expressed as a function of period lengthening $\tilde{T}_{eq}/T_{eq}$ for building different aspect ratios (h/r_θ) and embedment ratios (e/r_u).

Since actual soil/foundation conditions differ from those assumed in the development of Figure 7, guidance is needed in applying this relationship to realistic conditions. This is provided in the following sub-sections.

3.3.1 Effect of Non-Uniform Soil

Gazetas (1991) provides solutions for the impedance of rigid foundations overlying soil for which the shear stiffness increases with depth according to prescribed functions. The damping components of these solutions are plotted in Figure 8 in terms of the β_u and β_θ coefficients defined in Eq. 9. Also plotted for comparative purposes are the halfspace solutions. Damping values for non-uniform profiles are plotted for a zero hysteretic damping condition (radiation damping only). In Figure 8 the normalizing shear modulus and shear wave velocity are the values at the ground surface (G_0 and V_{s0}, respectively).

Figure 8 shows that the radiation damping in translation for a non-uniform profile is less than that for a halfspace at low frequencies. For rotation, a small reduction can occur at low frequencies, but the effect is less significant than for translation. At large frequencies, the radiation damping for non-uniform profiles exceeds that for the halfspace.

The low-frequency reduction in damping is due to reflections of body waves emanating from the foundation; the frequency dependence of the reduction is related to the depth over which the shear modulus increases relative to wavelength. For short wavelengths (low T) body waves, the non-uniform soil medium is "seen" as being effectively uniform, whereas long wavelength (large T) body waves "see" a much more non-uniform medium and wave transmission into the medium is impeded. The increase of radiation damping at high frequencies is due to the higher V_s of the non-uniform profiles at depth as compared to the velocity of the halfspace model (for which V_s was taken as V_{s0}).

An extreme case of soil non-uniformity is a finite soil layer of thickness H overlying a rigid base. In this case, soil damping cannot occur for periods larger than the fundamental site period, $T_s = 4H/V_s$.

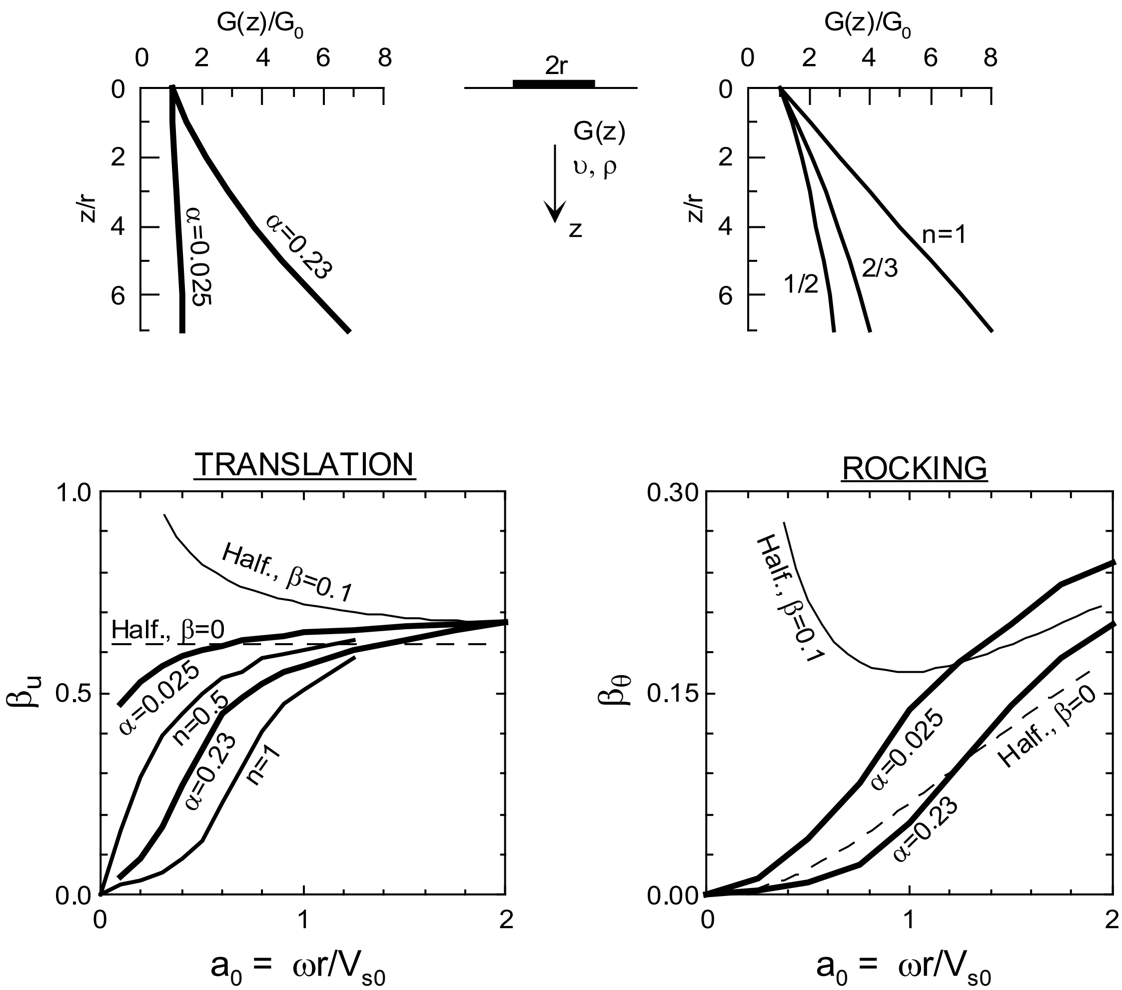

Figure 8. Foundation damping factors for halfspace with and without hysteretic damping (Veletsos and Verbic, 1973) and for soil profiles with indicated shear modulus profiles and no hysteretic damping (Gazetas, 1991).

Guidelines for practical application of the above results is summarized below:

- For translational damping, profile non-uniformity is not significant for $a_0=\omega r/V_s > 1$. Case history studies by Stewart et al. (1999) suggest that inertial soil-structure interaction is generally not important for $h/(V_sT) < 0.1$. Hence, for sites where SSI is important, profile non-uniformity need not be considered if $h/r < 2\pi h/(V_sT)$ or $V_sT/r < 2\pi$. The condition stated by the inequality is generally satisfied for sites having significant inertial SSI if $h/r < 2/3$, which is often the case for short-period buildings. Accordingly, it is often justified to treat the non-uniform soil as a halfspace, taking the halfspace velocity as the in situ value immediately below the foundation.
- Rotational damping for a non-uniform profile can generally be reasonably well estimated by a halfspace model, with the halfspace velocity taken as the in situ value immediately below the foundation.
- For sites with a finite soil layer overlying a very stiff material, foundation damping should be neglected for periods greater than the site period.

3.3.2 Effect of Embedment

Foundation embedment refers to a foundation base slab that is positioned at a lower elevation than the surrounding ground, which will usually occur when buildings have a basement. The impedance of embedded foundations differs from that of shallow foundations in several important ways. First, the static stiffness of embedded foundations is increased. Secondly, embedded foundations can produce much larger damping due to the greater foundation-soil contact area.

An approximate and generally conservative approach for estimating the damping of embedded foundations consists of using the increased static stiffness terms coupled with ordinary β_u and β_θ factors for surface foundations (i.e., Figure 6). This approach has been found to provide reasonable estimates of observed foundation damping in actual structures for embedment ratios $e/r_u < 0.5$ (Stewart et al., 1999). As short period structures are seldom deeply embedded, this approximate approach will often suffice for practical applications. For more deeply embedded foundations, alternative formulations can be used such as Bielak (1975) or Apsel and Luco (1987). The results shown in Figure 7 (right side) are based on this approximate approach.

3.3.3 Effect of Foundation Shape

The impedance function model described in Section 3.1 is based on representing foundations of arbitrary shape as equivalent circular mats through the use of radius terms r_u and r_θ (Eq. 8). The adequacy of this assumption for oblong foundations was investigated by Dobry and Gazetas (1986), who found that the use of equivalent circular mats is acceptable for aspect ratios less than 4:1, with the notable exception of dashpot coefficients in the rotation mode. For that condition, the translational damping is underestimated at low frequencies. This effect was neglected in the development of Figure 7, which is conservative.

3.3.4 Effect of Foundation Non-Rigidity

This section addresses flexibility in the foundation structural system (i.e., the base mat, or assemblage of a base mat and grade beams/footings). The foundation flexibility referred to here is not associated with the soil.

Impedance functions for flexible circular foundation slabs supporting shear walls have been evaluated for a number of wall configurations, including: (1) rigid core walls (Iguchi and Luco, 1986), (2) thin perimeter walls (Liou and Huang, 1994), and (3) rigid concentric interior and perimeter walls (Riggs and Waas, 1985). Those studies focused on the effects of foundation flexibility on rotation impedance; the horizontal impedance of flexible and rigid foundations are similar (Liou and Huang, 1994). Foundation flexibility effects on rotation impedance were found to be most significant for a rigid central core with no perimeter walls. For this case, the flexible foundation has significantly less stiffness and damping than the rigid foundation. The reductions are most significant for narrow central cores and large deviations between soil and foundation slab rigidity.

Significant additional work remains to be done on foundation flexibility effects on impedance functions because the existing research generally has investigated wall/slab configurations that are seldom encountered in practice for building structures. Nonetheless, based on the available studies and engineering judgment, the following preliminary recommendations were developed:

- The rigid foundation assumption is probably generally acceptable for the analysis of damping associated with horizontal vibrations of reasonably stiff, inter-connected foundation systems.

- For buildings with continuous shear walls or braced frames around the building perimeter, and continuous footing or mat foundations beneath these walls, the rigid foundation approximation can be used to provide a reasonable estimate of damping from rotation vibrations. In this case, the effective foundation radius (r_θ) would be calculated using the full building dimensions. This recommendation also applies if continuous basement walls are present around the building perimeter. This case is referred to as *stiff rotational coupling*.

- For buildings with a core of shear walls within the building, but no shear walls outside of this core, a conservative estimate of foundation damping can be obtained by calculating the effective foundation radius (r_θ) using the dimensions of the wall foundations (which, in this case, would be smaller than the overall building plan dimensions). This is an example of *soft rotational coupling* between the shear walls and other load bearing elements.

- For buildings with distributed shear walls at various locations around the building plan, the key issues are (1) the rotational stiffness of the building system as a whole (i.e., does the building tend to rotate as a single rigid block due to significant rotational stiffness coupling between adjacent elements, or do individual vertical components such as shear walls rotate independently of each other?), and (2) the degree to which destructive interference occurs between waves emanating from rotation of distinct foundation components.

In most cases, rotational coupling between vertical components is limited. In such cases, when foundation elements are widely spaced, the destructive interference would be small, and from a conceptual standpoint, it should be possible to evaluate the effective foundation system moment of inertia ($I_{f,eff}$) by assuming the walls act independently, as follows:

$$I_{f,eff} = \sum_i I_{f,i} \tag{13}$$

where $I_{f,i}$ represents the moment of inertia of an individual wall foundation. The effective foundation system radius ($r_{\theta,eff}$) for rotation would then be calculated using $I_{f,eff}$ in Eq. 8. This is an example of *soft rotational coupling without destructive interference*. However, when foundations are more closely spaced, destructive interference will occur and the above formulation may be unconservative. Unfortunately, this topic has not been researched, and thus what footing separation distances constitute "close" and "widely spaced" is unknown, which in turn precludes the development of recommendations for the analysis of rotation damping for distributed walls.

If rotational stiffness coupling between vertical elements is large (i.e., they tend to rotate as a rigid unit, e.g., because of deep spandrel beams between adjacent shear walls), but the vertical elements have independent footings, then the building has what is referred to as *intermediate rotational coupling*. In this case, the moment of inertia of the coupled elements can be estimated as

$$I_{f,i} = \sum_{j=1}^{M} A_j y_j^2 + \sum_{j=1}^{M} I_{f,j} \tag{14}$$

where $I_{f,i}$ = effective moment of inertia of j=1 to M coupled elements, A_j = area of footing j, y_j = normal distance from the centroid of the *jth* footing to the rotational axis of the coupled elements, and $I_{f,j}$ = moment of inertia of footing j. If the vertical elements for the entire building have intermediate rotational coupling, then $I_{f,i}$ from Eq. 14 is the effective moment of inertia for the foundation system as a whole. If the intermediate rotational coupling only occurs between selected vertical elements, then $I_{f,i}$ from Eq. 14 represents one contribution to the overall effective foundation moment of inertia, which can be calculated in consideration of all of the elements using Eq. 13 (with due consideration of potential destructive interference effects).

For buildings with only moment resisting frames (no walls or braced frames), foundation rotation is not likely to be significant, and hence foundation flexibility effects on rotation damping are also likely insignificant.

3.4 RECOMMENDED PROCEDURE

Step 1: Evaluate effective foundation radii using Eq. 8. The foundation area (A_f) for use in Eq. 8 is the full plan area if foundation elements are interconnected. The evaluation of an appropriate moment of inertial (I_f) is discussed in Section 3.3.4. Determine the foundation embedment, *e*, if applicable.

Step 2: Evaluate effective structure height, *h*, which is taken as the full height of the building for one story structures, and as the vertical distance from the foundation to the centroid of the first mode shape for multi-story structures. In the later case, *h* can often be well approximated as 70% of the total structure height.

Step 3: Evaluate the period lengthening ratio for the structure using the site-specific structural model developed for nonlinear pushover analyses. See Section 3.2 for details.

Step 4: Evaluate the initial fixed base damping ratio for the structure (β_i), which is often taken as 5%.

Step 5: Using Figure 7 and the guidelines in Section 3.3, estimate foundation damping (β_f) based on $\tilde{T}_{eq} / T_{eq}$, e/r_u, and h/r_θ..

Step 6: Evaluate the flexible base damping ratio (β_0) from β_f, β_i, and $\tilde{T}_{eq} / T_{eq}$ using Eq. 10.

Step 7: Evaluate the effect on spectral ordinates of the change in damping ratio from β_i to β_o using established models (e.g., Eq. 8-10 of *ATC-40*; model for use in *FEMA 440* remains under investigation).

4.0 CONCLUSIONS

In this paper, we have presented sets of recommendations for incorporating the effects of kinematic soil-structure interaction and foundation damping into assessments of seismic demand for use in nonlinear static analysis procedures for building structures. These effects generally decrease the seismic demand relative to what would be used in current practice, which is based on 5% structural damping and equivalent foundation and free-field motions. The demand reduction is greatest at short periods. The recommended procedures are summarized in Sections 2.4 and 3.4, respectively.

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